Seismic Waves

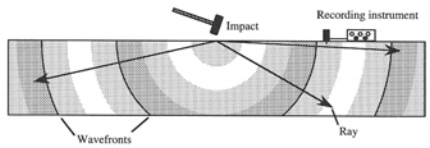


Figure 2-1 A hammer blow creates a disturbance that propagates through a homogeneous granite as seismic waves. Wavefronts and rays are indicated.

Wave Terminology: Wavelength, Amplitude

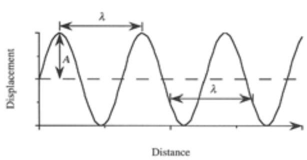


Figure 2-2 Pattern of particle displacements during passage of wave created by hammer blow illustrated in Figure 2-1. λ = wavelength; A is amplitude of maximum particle displacement from normal position.

Velocity, Frequency & Wavelength

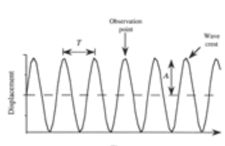


Figure 2-3 Motion of a single water particle in lake or a particle in granite as waves pass by. T (wave period) is the time for two successive wave crests to pass the observation point.

point every tenth of a second. The frequency, or repetitions per second, is then 10. It should be clear that frequency and period are related in the form

$$f = \frac{1}{T} \qquad (2\text{-}1)$$

The unit of frequency is the *hertz* (Hz) which is the number of repetitions per second.

If we refer to lake ripples once again, it is evident that the speed with which the ripple crest or wavefront is moving can be determined by noting the time it takes the wavefront to traverse a known distance

$$V = \frac{distance}{time} \quad (\mathrm{ms^{-1}}) \qquad (2\text{-}2)$$

Similarly, speed can be determined if we know the wavelength and the frequency

$$V = f\lambda \quad (\mathrm{ms^{-1}}) \qquad (2\text{-}3)$$

Stress and Strain: Hooke's Law Young's Modulus

If a Hookean material is subjected to a uniaxial compression or tension, the linear relationship between the applied stress σ and resulting strain ε is given by

$$\sigma = E\varepsilon \qquad (2\text{-}4)$$

where the constant of proportionality E is termed *Young's modulus*. As this constant directly relates resultant strain to a given stress, it seems probable that rocks with different values for E may we!! have different velocities.

Elongation ε is the change in length of a line in its final or deformed state divided by its original length

$$\varepsilon = \frac{l_f - l_o}{l_o} = \frac{\Delta l}{l_o} \qquad (2\text{-}5)$$

Stress and Strain: Poisson's Ratio, Bulk Modulus

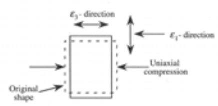


Figure 2-4 A uniaxial compression produces a positive elongation $+\varepsilon_1$ and a negative elongation $-\varepsilon_3$. The ratio of these two strains is referred to as Poisson's ratio μ.

will shorten in the direction of the applied stress but at the same time will lengthen in a direction at right angles to the compression. Elongations can be measured for each direction, and their ratio is referred to as *Poisson's ratio* μ:

$$\mu = -\frac{\varepsilon_1}{\varepsilon_3} \quad \text{where } \mu \le 0.5 \qquad (2\text{-}6)$$

Two additional elastic coefficients also are important. Given an isotropic material, subject it to a general pressure change (Fig. 2-5). A volume change will occur, and this change from original volume V_o to final volume V_f, when compared to the pressure change that induced it, is termed the *bulk modulus* K:

$$K = \frac{\Delta P}{\Delta} \quad \text{where} \quad \Delta = \frac{V_f - V_o}{V_o} \qquad (2\text{-}7)$$

The bulk modulus is a measure of the incompressibility of the material.

Stress and Strain: Bulk Modulus, Rigidity Modulus

Finally, it is possible to deform a solid by simple shear (Fig. 2-6). In this case a shear

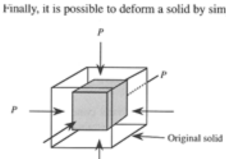


Figure 2-5 A change in volume (reduction) produced by a change in pressure (increase). The ratio of the pressure change to the volume change is a measure of the incompressibility of the material and is known as the bulk modulus K.

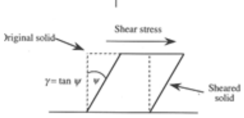


Figure 2-6 A measure of the shearing resistance of a material is the ratio of the shear stress to the shear strain, which is termed the rigidity modulus G. The angular shear is ψ.

Stress and Strain: Equivalence

strain γ is induced by applying a shear stress σ_r. The ratio of these quantities is the *rigidity modulus G*:

$$G = \frac{\sigma_r}{\gamma} \tag{2-8}$$

If we pause for a moment to consider Hookean materials, it seems reasonable to propose that the quantities E, μ, K, and G reasonably thoroughly define elastic behavior and, therefore, must be important factors in governing the speeds with which disturbances travel through Earth materials. Fortunately, values for all of these quantities can be determined in the laboratory. Some representative values for common rock types are given in Table 2-1. As you might guess, these four quantities are not mutually independent. Specifically, G and K can be defined in terms of E and μ:

$$G = \frac{E}{2(1 + \mu)} \tag{2-9}$$

$$K = \frac{E}{3(1 - 2\mu)} \tag{2-10}$$

Elastic Coefficients

TABLE 2-1 Elastic Coefficients and Seismic Velocities for Selected Common Rocks

Rock Type	Density ρ	Young's Modulus E	Poisson's Ratio μ	Vp (m/s)	Vs (m/s)	Vp/Vs	Vs as %Vp
Shale (AZ)	**2.67**	**0.120**	**0.040**	2124	1470	1.44	69.22%
Siltstone (CO)	**2.50**	**0.130**	**0.120**	2319	1524	1.52	65.71%
Limestone (PA)	**2.71**	**0.337**	**0.156**	3633	2319	1.57	63.84%
Limestone (AZ)	**2.44**	**0.170**	**0.180**	2750	1718	1.60	62.47%
Quartzite (MT)	**2.66**	**0.636**	**0.115**	4965	3274	1.52	65.96%
Sandstone (WY)	**2.28**	**0.140**	**0.060**	2488	1702	1.46	68.42%
Slate (MA)	**2.67**	**0.487**	**0.115**	4336	2860	1.52	65.96%
Schist (MA)	**2.70**	**0.544**	**0.181**	4680	2921	1.60	62.41%
Schist (CO)	**2.70**	**0.680**	**0.200**	5290	3239	1.63	61.24%
Gneiss (MA)	**2.64**	**0.255**	**0.146**	3189	2053	1.55	64.38%
Marble (MD)	**2.87**	**0.717**	**0.270**	5587	3136	1.78	56.13%
Marble (VT)	**2.71**	**0.343**	**0.141**	3643	2355	1.55	64.65%
Granite (MA)	**2.66**	**0.416**	**0.055**	3967	2722	1.46	68.62%
Granite (MA)	**2.65**	**0.354**	**0.096**	3693	2469	1.50	66.85%
Gabbro (PA)	**3.05**	**0.727**	**0.162**	5043	3203	1.57	63.51%
Diabase (ME)	**2.96**	**1.020**	**0.271**	6569	3682	1.78	56.05%
Basalt (OR)	**2.74**	**0.630**	**0.220**	5124	3070	1.67	59.91%
Andesite (ID)	**2.57**	**0.540**	**0.180**	4776	2984	1.60	62.47%
Tuff (OR)	**1.45**	**0.014**	**0.110**	996	659	1.51	66.20%

Units for Young's modulus are $(N/m^2) \times 10^{11}$. Velocities computed from ρ, E, and μ. Values selected from Press (1966, pp. 97–173).

Components of Ground Motion induced by P-Waves

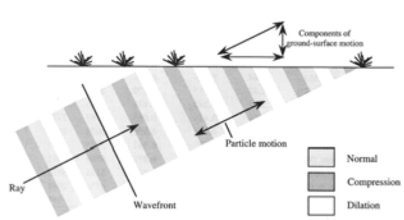
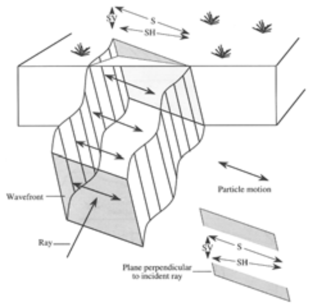
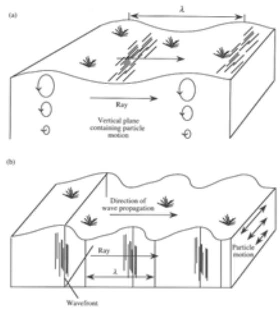


Figure 2-7 P-wave incident on surface. Particle motion is parallel to direction of travel. Since the wavefront plane is canted to the horizontal plane, ground-surface motion will have horizontal and vertical components.

Components of Ground Motion induced by S-Waves



Components of Ground Motion induced by Surface Waves



Velocity varies with Elastic Cofficients

The velocities of P- and S-waves are

$$V_P = \sqrt{\frac{K + 4/3\,G}{\rho}} = \sqrt{\frac{E}{\rho}\frac{(1-\mu)}{(1-2\mu)(1+\mu)}} \quad (2\text{-}11)$$

and

$$V_S = \sqrt{\frac{G}{\rho}} = \sqrt{\frac{E}{\rho}\frac{1}{2(1+\mu)}} \quad (2\text{-}12)$$

If we note that the bulk modulus K and the rigidity modulus G are always positive and recall that Poisson's ratio μ is less than or equal to 0.5, then it is evident that the velocity of P-waves always must be greater than that of S-waves by a considerable factor. Establishing the actual ratio consists simply of applying the preceding equations and simplifying

$$\frac{V_P}{V_S} = \sqrt{\frac{1-\mu}{1/2-\mu}} \quad (2\text{-}13)$$

Because G is equal to zero for liquids, the velocity of S-waves in liquids goes to zero. In other words shear waves cannot be propagated by liquids. It is this fact and the observation that direct S-waves are not recorded in the earth's "shadow zone" that constitute a primary line of evidence for a liquid outer core.

P-Wave Velocities of Earth Matierals

TABLE 2-2 Representative P-wave Velocities

Unconsolidated Materials		Consolidated Materials		Other	
Weathered layer	300–900	Granite	5000–6000	Water	1400–1600
Soil	250–600	Basalt	5400–6400	Air	331.5
Alluvium	500–2000	Metamorphic rocks	3500–7000		
Clay	1100–2500	Sandstone and shale	2000–4500		
Sand		Limestone	2000–6000		
Unsaturated	200–1000				
Saturated	800–2200				
Sand and gravel					
Unsaturated	400–500				
Saturated	500–1500				
Glacial till					
Unsaturated	400–1000				
Saturated	1700				
Compacted	1200–2100				

Velocities in m/s

Velocity ranges compiled from Press (1966, p. 195–218).

Rules of Thumb for P-wave Velocities

- Unsaturated sediments have lower values than saturated sediments.
- Unconsolidated sediments have lower values than consolidated sediments.
- Velocities are very similar in saturated, unconsolidated sediments.
- Weathered rocks have lower values than similar rocks that are unweathered.
- Fractured rocks have lower values than similar rocks that are unfractured.

Rules of Thumb for S-wave and Surface Wave Velocities

In some field studies we also will want to estimate velocities for S-waves and Rayleigh waves. Love waves typically are not generated during exploration work, so we have little interest in estimating their velocities. General rules of thumb for such estimates are

$$
\begin{aligned}
V_S &= 0.6\,V_P && \text{for crystalline rocks} \\
V_S &= 0.5\,V_P && \text{for sedimentary rocks} \\
V_S &= 0.4\,V_P && \text{for soils and unconsolidated materials} \\
V_R &= 0.9\,V_S &&
\end{aligned}
\qquad (2\text{-}14)
$$

Propagation of Wavefront

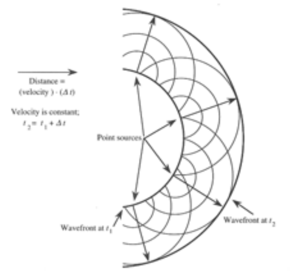


Figure 2-10 Applying Huygens' principle to determine the position of the wavefront at t_2 after an interval of time Δt. Given the position of a wavefront at time t_1 and applying Huygens' principle, the position of the wavefront at time t_2 is determined.

Angle of Incidence & Angle of Reflection

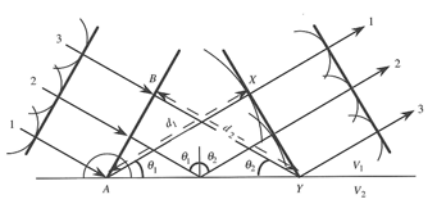


Figure 2-11 Using Huygens' principle to demonstrate that the angle of incidence equals the angle of reflection ($\theta_1 = \theta_2$). Analysis detailed in the text.

Angle of Incidence & Angle of Reflection

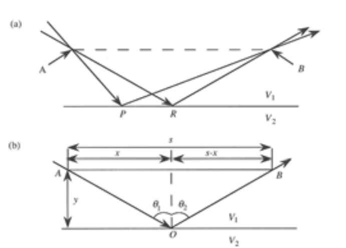


Figure 2-12 Using Fermat's principle to demonstrate that the angle of incidence equals the angle of reflection ($\theta_1 = \theta_2$). (a) Which path takes the least time? (b) Symbols used for analysis in text.

Angle of Incidence & Angle of Reflection

By referring to Figure 2-12(b), we can state that the time (path distance/velocity) for a ray to travel from A through some point O to B is

$$t = \frac{(x^2 + y^2)^{1/2}}{V_1} + \frac{((s - x)^2 + y^2)^{1/2}}{V_1} \quad (2\text{-}15)$$

In order to determine the minimum value of time t, we take the first derivative of the function and set it equal to zero:

$$\frac{dt}{dx} = \frac{x}{V_1(x^2 + y^2)^{1/2}} - \frac{(s - x)}{V_1((s - x)^2 + y^2)^{1/2}} = 0 \quad (2\text{-}16)$$

Using the relationships

$$\sin\theta_1 = \frac{x}{(x^2 + y^2)^{1/2}} \quad \text{and} \quad \sin\theta_2 = \frac{(s - x)}{((s - x)^2 + y^2)^{1/2}} \quad (2\text{-}17)$$

we see that

$$\frac{\sin\theta_1}{V_1} - \frac{\sin\theta_2}{V_1} = 0 \quad \text{and, therefore,} \quad \theta_1 = \theta_2 \quad (2\text{-}18)$$

Thus, the path for which the time of travel is least is the one for which the angle of incidence equals the angle of reflection.

Angle of Incidence & Angle of Refraction

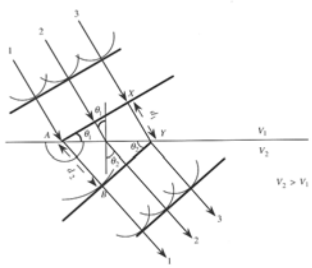


Figure 2-13 Using Huygens' principle to demonstrate the relationship between the angles of incidence and refraction. Analysis detailed in the text.

Angle of Incidence & Angle of Refraction

The construction in Figure 2-13 is essentially the same as in Figure 2-11, except here we deal with refracted rays. When ray 1 arrives at point A, it creates a disturbance in the material with velocity V_2. The disturbance spreads outward in this layer and will travel a distance d_2 during the time t_1 it takes ray 3 to travel from point X to point Y (a distance d_1). We construct the position of the new wavefront at the moment ray 3 arrives at Y by drawing a line connecting Y and B (which is the tangent to the wavelet with radius d_2 that was generated at A). Because

$$\sin\theta_2 = \frac{d_2}{AY} \quad \text{and} \quad \sin\theta_1 = \frac{d_1}{AY} \quad (2\text{-}19)$$

$$d_1 = t_1V_1 \quad \text{and} \quad d_2 = t_1V_2 \quad (2\text{-}20)$$

it holds that

$$AY = \frac{t_1V_1}{\sin\theta_1} = \frac{t_1V_2}{\sin\theta_2} \quad \text{and} \quad \frac{\sin\theta_1}{\sin\theta_2} = \frac{V_1}{V_2} \quad (2\text{-}21)$$

Angle of Incidence & Angle of Refraction

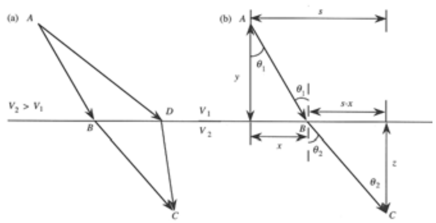


Figure 2-14 Using Fermat's principle to demonstrate the relationship between the angles of incidence and refraction. (a) What are the distinguishing features of the path of least time (*ABC*) as opposed to others (*ADC*)? (b) Symbols for analysis used in text.

Angle of Incidence & Angle of Refraction

When applying Fermat's principle, we can ask the same sort of questions that we did for the reflection problem. When examining Figure 2-14(a), we would like to know what distinguishes the path of least time of a ray traveling from *A* to *C* as it passes through an interface separating materials of differing velocities. Once again we write an equation expressing the time it takes for a ray to travel from *A* through *B* to *C*:

$$t = \frac{(x^2 + y^2)^{1/2}}{V_1} + \frac{((s - x)^2 + z^2)^{1/2}}{V_2} \qquad (2\text{-}22)$$

Continuing, we take the derivative and arrive at the expression:

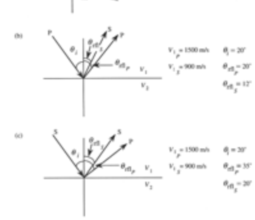
$$\frac{dt}{dx} = \frac{x}{V_1(x^2 + y^2)^{1/2}} - \frac{(s - x)}{V_2((s - x)^2 + z^2)^{1/2}} = 0 \qquad (2\text{-}23)$$

In taking the derivative, note that z and y are constants as they retain the same value regardless of where B is placed along the interface. As before, we can use identities to arrive at the final form relating angles and velocities:

$$\sin\theta_1 = \frac{x}{(x^2 + y^2)^{1/2}} \quad \text{and} \quad \sin\theta_2 = \frac{(s - x)}{((s - x)^2 + z^2)^{1/2}} \qquad (2\text{-}24)$$

$$\frac{\sin\theta_1}{V_1} - \frac{\sin\theta_2}{V_2} = 0 \quad \text{and, therefore,} \quad \frac{\sin\theta_1}{\sin\theta_2} = \frac{V_1}{V_2} \qquad (2\text{-}25)$$

Reflections and Refractions



Reflections and Refractions